

NPV Model

Variables

NPV : [1.0 USD2014] net present value

t : [1.0 year] time

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PV : [1.0 USD2014] present value

C : [1.0 USD2014] cash flow

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C_r : [1.0 USD2014 / year] cash flow rate

t : [1.0 year] time

C : [1.0 USD2014] cash flow

r : [1.0 1 / year] interest rate

Problem Set Up

Let's assume that we want the Net Present Value (NPV) to be \$10 Million. NPV can be expressed by:

$$NPV = \sum_{i=0}^N PV_i$$

.

Let's assume that the time periods of the payments are not equal but that the PV_i are equal and that there are 3 payments. PV can be expressed by

$$PV = Ce^{-rt}$$

.

With these assumptions we can get rid of the i subscript and claim that

$$NPV \leq 3PV$$

Now the problem becomes solving for the length of each time period such that each PV is equal. Let's assume that the same payment, C , is made at each payment period and that C is given. This allows us to write

$$C \leq PVe^{rt_i}$$

Let's assume that t_0 is the time of evaluation of the NPV and that t_i is when every payment is made and every PV evaluated. This means that

$$\begin{bmatrix} \Delta t_1 = t_1 - t_0 \\ \Delta t_2 = t_2 - t_1 \\ \Delta t_3 = t_3 - t_2 \end{bmatrix}$$

If there is a cash flow rate, C_r then,

$$C \leq C_r \Delta t$$

Constraints and Objective

$$\begin{aligned} & \text{minimize} \quad \frac{1}{NPV} \left[\frac{1}{\text{USD}} \right] \\ & \text{subject to} \quad \frac{1}{NPV} \left[\text{USD} \right] \leq 3PV \left[\text{USD} \right] \\ & \quad \vec{C} \geq \left[0.04167PVr^4t_{(0)}^4 + 0.1667PVr^3t_{(0)}^3 + 0.5PVr^2t_{(0)}^2 + PV + PVrt_{(0)} \right] \left[\text{USD} \right] \quad 0.04167PVr^4t_{(0)}^4 \\ & \quad \vec{t} \geq \left[\frac{C_{(0)}}{C_r} \left[\text{year} \right] \quad \frac{C_{(1)}}{C_r} + t_{(0)} \left[\text{year} \right] \quad \frac{C_{(2)}}{C_r} + t_{(1)} \left[\text{year} \right] \right] \end{aligned}$$

Cost — 3.333e-08 [1/USD]

| Free Variables | Value | Units | Description |
|----------------|-------------------------------|----------|-------------------|
| NPV | 3.001e+07 | $[USD]$ | net present value |
| PV | 1e+07 | $[USD]$ | present value |
| \vec{C} | [1.3e+07 1.89e+07 5.36e+07] | $[USD]$ | cash flow |
| \vec{t} | [2.59 6.37 17.1] | $[year]$ | time |

| Constants | Value | Units | Description |
|-----------|-------|--------------|----------------|
| C_r | 5e+06 | $[USD/year]$ | cash flow rate |
| r | 0.1 | $[1/year]$ | interest rate |

| Sensitivities | Value | Units | Description |
|---------------|-------|-------|----------------|
| r | 1 | | interest rate |
| C_r | -1 | | cash flow rate |